

This is to solve the Sage portion of Chapter 6 of

"A journey through the realm of numbers: from quadratic equations to quadratic reciprocity."

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In [1]: # Distances between perfect powers.
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```
In [2]: def perfect(k):
         top = 10^k
         lastb = floor(sqrt(top))
         powerlist = [1]
         for b in range(2, lastb+1):
             if not(b in powerlist):
                 laste = floor(log(top,b))
                 newentries = [b^k for k in range(2, laste+1)]
                 powerlist = powerlist+newentries
         return sorted(powerlist)
```

```
In [3]: perfect(2)
```

```
Out[3]: [1, 4, 8, 9, 16, 25, 27, 32, 36, 49, 64, 81, 100]
```

```
In [4]: def catalan(k):
         powerlist = perfect(k)
         count = len(powerlist)
         print(all(powerlist[l+1]>powerlist[l]+1
                   for l in range(3, count-1)))
```

```
In [5]: catalan(4) # is Catalan true up to 10000?
```

```
True
```

```
In [6]: catalan(6) # is Catalan true up to 10^6?
```

```
True
```

```
In [7]: catalan(9) # is Catalan true up to 10^9?
```

```
True
```

```
In [8]: # Since perfect powers grow very quickly, we can easily check
         # Catalan's conjecture up to 10^9.
```

```
In [9]: def pillai(k):
         powerlist = perfect(k)
         count = len(powerlist)
         for l in range(count-1):
             print(powerlist[l], powerlist[l+1] - powerlist[l])
```

```
In [10]: pillai(2)
```

```
1 3
4 4
8 1
9 7
16 9
25 2
27 5
32 4
36 13
49 15
64 17
81 19
```

```
In [11]: def pillai2(k):
          powerlist = perfect(k)
          stages = [[p for p in powerlist
                     if 10^n <= p <= 10^(n+1)]
                   for n in range(k)]
          # this is not handling the gap from 9 to 16
          # correctly (since 10 isn't yet a power) but
          # as this difference is not a smallest one it is o.k.
          for n in range(k):
              count = len(stages[n])
              difflist = [stages[n][a+1] - stages[n][a]
                         for a in range(count-1)]
              print(min(difflist))
```

```
In [12]: pillai2(9)
```

```
1
2
3
10
7
17
15
37
19
```

```
In [ ]:
```