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Functional analysis, spectral theory, and applications. (English) Zbl 06732355

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The present book is different from the usual textbooks on functional analysis: it does not only cover the basic material but also a number of advanced topics which cannot be found in many other books on the subject.

The book consists of fourteen chapters, whose contents will be briefly described in the following. The first chapter discusses some motivations for the study of functional analysis, such as applications to group representations, PDEs and number theory (the prime number theorem, see below).

The second chapter contains the standard material on norms and Banach spaces as well as bounded operators and functionals. Some advanced results such as the Mazur-Ulam theorem and the Stone-Weierstraß theorem are also included (there is a whole section on spaces of continuous functions). At the end of this chapter, some applications to Volterra and Sturm-Liouville equations are discussed.

Chapter 3 deals with Hilbert spaces and Fourier series (both on the d-dimensional torus and on general compact abelian groups). There is also a whole section on group actions and unitary representations.

The fourth chapter contains the uniform boundedness principle, the open mapping and the closed graph theorem.

Chapter 5 discusses Sobolev spaces and applications to Dirichlet boundary value problems.

Chapter 6 deals with compact self-adjoint operators (in particular with their spectral theory). There is also a section on trace-class operators and one on eigenfunctions of the Laplace operator (including a proof of Weyl's law).

The seventh chapter concerns dual spaces. In the first section the authors discuss the Hahn-Banach theorem and its consequences. The second section deals with some nonstandard material such as Banach limits, amenable groups and the Banach-Tarski paradox. Section 3 contains the description of the duals of L^p-spaces and the Riesz-Thorin interpolation theorem. The fourth section deals with duals of $C(K)$-spaces.

Locally convex spaces are treated in Chapter 8. The authors first discuss the weak- and the weak*-topology (including of course the Banach-Alaoglu theorem). Then they present some applications of weak*-compactness (equidistributed sequences; elliptic regularity of the Laplace operator). In Section 8.3. they introduce the strong and the weak operator topology, before defining general locally convex spaces in Section 8.4. This is followed by a short section on distributions. Section 8.6. then deals with convex sets and extreme points (including the Krein-Milman and also the Choquet theorem).

In Chapter 9 the authors treat the spectral theory and functional calculus for unitary operators, the Fourier transform, and also the spectral theory of unitary flows (unitary representations of $\mathbb{R}^d$).

Chapter 10 concerns amenable groups and the property (T), a topic which is normally not found in textbooks on functional analysis. The chapter even contains a section on the explicit construction of expander graphs.

Chapter 11 deals with Banach algebras, in particular with C^*-algebras and the Gelfand transform.

In Chapter 12 the authors introduce the spectral theory and functional calculus for bounded self-adjoint operators. As a particular example, the Laplace operator on a regular tree is discussed. Chapter 13 then deals with the spectral theorem for unbounded self-adjoint operators.

The last chapter is a particular highlight of the book. It touches a topic which is rarely ever mentioned in other functional analysis books, namely the importance of functional analysis in analytic number theory. Concretely, the authors present a proof of the prime number theorem using Banach algebras, following T. Tao's blog.

The book also contains two appendices (one on set theory and topology, the other on measure theory)

and more than 400 exercises throughout the text. For about half of them hints are given at the end of the book. The text is suitbale for self-study as well as for the preparation of lectures and seminars.

In summary, this is a highly recommendable book for students and researcher alike who are interested in functional analysis and its broad applications.

Reviewer: Jan-David Hardtke (Berlin)

MSC:

46-01 Textbooks (functional analysis)
47-01 Textbooks (operator theory)
46B10 Duality and reflexivity in normed spaces
46C05 Hilbert and pre-Hilbert spaces: geometry and topology
46A03 General theory of locally convex spaces
46E15 Banach spaces of continuous, differentiable or analytic functions
46E30 Spaces of measurable functions
46E35 Sobolev spaces and other spaces of "smooth" functions, embedding theorems, trace theorems
46L05 General theory of C^*-algebras
47A05 General theory of linear operators
47A10 Spectrum and resolvent of linear operators
47A60 Functional calculus of operators
47B15 Hermitian and normal operators
47B25 Symmetric and selfadjoint operators (unbounded)
43A07 Means on groups, semigroups, etc.; amenable groups

Keywords:

Banach spaces; Hilbert spaces; spectral theory; functional calculus; Banach algebras; amenable groups; prime number theorem

Full Text: DOI