Errata for
'Ergodic Theory with a view towards Number Theory'
by Einsiedler and Ward.

The authors reiterate their thanks to the many people who sent in comments on early drafts of the book, and add to their number Beat Jäckle . This file contains errata and some other small changes. We thank Lovy Singhal for pointing out the missing assumption in Exercise 2.1.7 on page 20, Constantin Kogler for pointing out problems in several places, Anurag Rao for raising the issue on page 206, Andreas Wieser for pointing out a missing hypothesis in Exercise 4.1.3, David Simmons and Anurag Rao for pointing out a problem with the statements of Lemma 11.24 and Theorem C.4, and Manuel Lüthi for correcting several cross-references.

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page 18, footnote. This should be $|i| \leqslant j$ rather than $|j| \leqslant k$.

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page 20, Exercise 2.1.7. Assume in addition that X is a compact metric space and that $\mathscr{B}$ is the Borel σ-algebra on X. Notice that T is still only assumed to be measurable and measure-preserving.

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page 33, formula after (2.9). First expression should be squared.

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page 36. First displayed equation should be $\leqslant$ rather than $<$.

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page 55. Displayed union after (2.33) should go from $k = 1$.

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page 62. Inline equation after Definition 2.42 should be $r_A(x)$ not $r_a(x)$.

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page 84, from line 6 the text should read as follows: However, for any fixed N the truncated function

$$g_N(x) = \begin{cases} g(x) & \text{if } g(x) \leqslant N; \\ 0 & \text{if not} \end{cases}$$

is in L^1_μ. Now

$$\int g_N \, \mathrm{d}\mu \geqslant \frac{1}{2\log 2} \sum_{a=1}^{N} \int_{1/(a+1)}^{1/a} a \, \mathrm{d}x = \frac{1}{2\log 2} \sum_{a=1}^{N} \frac{1}{a+1},$$

so $\int_0^1 g_N \, \mathrm{d}\mu \to \infty$ as $N \to \infty$.

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page 98. Second line of the three-line displayed formula should have exponent $n(j)$ not $n(j) + 1$.

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page 102. Exercise 4.1.3. As stated this requires the map T to be surjective; in general the system (X, T) needs to be replaced with (X_∞, T) where $X_\infty = \bigcap_{n \geqslant 0} T^n(X)$.

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page 104. Exercise 4.2.4(iii) should ask for $n \in \mathbb{Z}$ rather than $n \in \mathbb{N}$.

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page 130. Last line of page should read "Then $E \in \mathscr{A}$, so" (that is, omitting "and $\varepsilon\chi_E \leqslant E(f|\mathscr{A})$")

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page 146. Displayed equation in statement of Lemma 5.23 and in the second line of the proof should have $\overline{X}$ instead of X.

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page 147, line 13. Continue the sentence to say "and on which ϕ is defined."

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page 147, line 14. The σ-algebra $\mathscr{A}$ should be restricted to X'.

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page 147, line -10. The last $\mathscr{B}$ should be $\mathscr{B}_Y$.

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page 168, Exercise 6.5.3: the hypothesis that both systems are ergodic should be added.

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page 177. In order to have the hypothesis that the space is a Borel space available for the invertible extension, Sections 7.2.1 and 7.2.2 should be reversed, with the map in the new Section 7.2.1 (Reduction to Borel Probability Spaces) being defined into the space $\{0,1\}^{\mathbb{N}}$.

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page 194. In the first paragraph of Section 7.6.1, $f_1, f_2 \in L^\infty_\mu$ should be $f_1, f_2 \in L^\infty_{m_G}$, and in the displayed equation $U_{R_a \times R_a^2}(f_1 \otimes f_2)$ should be $U^n_{R_a \times R_a^2}(f_1 \otimes f_2)$.

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page 206, line 15 onwards. This should begin "$\mathscr{F}$ contains $L^\infty(Y)$ and is an algebra of functions". Then the following text should be added at the end of the paragraph: "To see that if $f \in L^\infty(Y)$ then $f \in \mathscr{F}$, let $\varepsilon > 0$ and suppose that $c_1, \dots, c_r \in \mathbb{C}$ are ε-dense in

$$\{z \in \mathbb{C} \mid |z| < \|f\|_\infty\}.$$

Noticing that $U^n_T f$ is constant on every $\mathscr{A}$-atom, it follows that the constant functions $g_j(x) = c_j$ for $x \in X$ and $j = 1, \dots, r$ satisfy the requirements of Definition 7.18."

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page 252. In the proof of Theorem 8.10 the reference to Section A.3 should be to Section B.7.

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page 255, second line. Reference to Section A.3 should be to Section B.7.

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page 261, last displayed equation. This should read

$$m_G\left(\bigcup_{j=1}^{K} B^G_{r_j} a_j\right) \leqslant \sum_{i=1}^{k} m_G(B^G_{3r_{j(i)}} a_{j(i)}) \leqslant C_G \sum_{i=1}^{k} m_G(B^G_{r_{j(i)}} a_{j(i)}).$$

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page 265, line -2: This should read 'Similarly, if $g' \notin B^G_{r+M}$ then...' (replacing $g' \in B^G_{r+M}$)

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page 306, after displayed equation in proof of Lemma 9.16: 'where $x = x + \mathrm{i}y$' should be 'where $z = x + \mathrm{i}y$'

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page 308. Figure 9.5: the two points identified in Cartesian coordinates should be in brackets.

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page 321. Notation in Lemma 9.23 changed from X_r to Y_r to avoid confusion with X_2.

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page 339. In the middle of the page, sentence beginning 'Then, as discussed above,' the variables x and x' need to vary with the parameter ℓ. Thus x and x' should be replaced in the next displayed equation and in the last but one displayed equation on the page by x_ℓ and x'_ℓ respectively.

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page 371, Lemma 11.24. Our thanks to David Simmons and Anurag Rao for pointing out a problem with this argument, which can be corrected as follows. The definition of the set $S(\alpha)$ should be changed to be: Define $S(\alpha)$ to be the set of $g \in G$ with the property that there exist sequences (n_k) in $\mathbb{N}$ and (h_k) in G such that $h_k \to g$ and $a_{n_k}^{-1} h_k a_{n_k} \to e$ as $k \to \infty$, where e is the identity element in G. Recall from the discussion of $\mathrm{SL}_2(\mathbb{R})$ as a closed linear group on p. 289 that the homomorphism $\phi : \mathrm{SL}_2(\mathbb{R}) \to \mathrm{GL}\left(\mathrm{Mat}_{22}(\mathbb{R})\right)$ defined by $(\phi(g))\,(v) = gvg^{-1}$ is a proper map. That is, the norm of $\phi(g_n)$ goes to infinity when (g_n) is a sequence that leaves compact subsets. Notice that $\mathrm{Mat}_{22}(\mathbb{R}) = \mathbb{R}I_2 \oplus \mathfrak{sl}_2(\mathbb{R})$ splits into a sum of two invariant subspaces on the first of which the action is trivial. Thus the norm of Ad_{g_n} goes to infinity; that is we can choose a sequence of vectors (v_n) in $\mathfrak{sl}_2(\mathbb{R})$ for which $\|v_n\| \to 0$ while $\|\mathrm{Ad}_{g_n}(v_n)\| = c > 0$ (where c is some fixed small constant chosen so that the exponential map is injective on the ball of radius $2c$). By exponentiating this sequence, and choosing an appropriate subsequence we get $h'_k = \exp(v_{n_k}) \to I_2$ and $h_k = g_{n_k} h_k g_{n_k}^{-1} \to u \neq I_2$ as $k \to \infty$. Since for large k the element h'_k is close to the identity, its eigenvalues are close to 1. The conjugated element $h_k = g_{n_k} h'_k g_{n_k}^{-1}$ has the same eigenvalues, so the limit element has 1 as its only eigenvalue — thus u is unipotent and non-trivial.

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page 427, last line. In the displayed equation, g_P should be x_P.

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page 431, statement of Theorem C.4. As written (2) is not correct unless the measure is assumed to be Radon (locally finite). For simplicity – and sufficient for our needs – please add the hypotheses metric and separable to the requirement that the group be locally compact in the statement of the theorem.

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page 463, Author index. Reference to 'atom' should be in the general index.

June 11, 2019