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★**Functional analysis, spectral theory, and applications.**
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This textbook on functional analysis consists of the following fourteen chapters: 1. Motivation; 2. Norms and Banach Spaces; 3. Hilbert Spaces, Fourier Series, Unitary Representations; 4. Uniform Boundedness and the Open Mapping Theorem; 5. Sobolev Spaces and Dirichlet's Boundary Problem; 6. Compact Self-Adjoint Operators, Laplace Eigenfunctions; 7. Dual Spaces; 8. Locally Convex Vector Spaces; 9. Unitary Operators and Flows, Fourier Transform; 10. Locally Compact Groups, Amenability, Property (T); 11. Banach Algebras and the Spectrum; 12. Spectral Theory and Functional Calculus; 13. Self-Adjoint and Symmetric Operators; 14. The Prime Number Theorem.

All chapters end with a very useful list of additional topics and suggestions for further reading. The book also contains an appendix on set theory and topology, another one on measure theory, and a third one with hints to about one half of the 402 exercises that are spread all over the text, with some of them named essential.

The authors are established researchers and know how to motivate and to explain. The book is carefully written and provides an interesting introduction to functional analysis with a wealth of both classical and more recent applications. Particular emphasis is on topological groups and unitary representations of such groups on a complex Hilbert space. There are some fairly advanced and diverse applications which cannot be found in other introductions to this area. For instance, the discussion of Každan's property (T) in Chapter 10 and of Tao's very recent Banach algebra approach to the prime number theorem in Chapter 14 clearly sets this book apart from classical textbooks on functional analysis such as the ones by John B. Conway [*A course in functional analysis*, second edition, Grad. Texts in Math., 96, Springer, New York, 1990; MR1070713] and Walter Rudin [*Functional analysis*, second edition, Internat. Ser. Pure Appl. Math., McGraw-Hill, New York, 1991; MR1157815]. As indicated by the titles of the chapters, there is a strong emphasis on the Hilbert space setting, and the authors manage to cover a number of interesting applications already in the first third of the book.

However, it is clear that not all aspects of a field as rich as functional analysis can be covered in a textbook of about 600 pages. For instance, Chapter 8 contains a discussion of the Banach-Alaoglu theorem on weak* compactness together with a nice application to the elliptic regularity for the Laplace operator, but the classical Krein-Šmulian theorem on weak* closed subsets of the dual of a Banach space or the remarkable results of Eberlein and Šmulian, and James on weakly compact subsets of a Banach space are beyond the scope of the monograph under review. We note that these results are either proved or at least mentioned in Conway's book.

Locally convex vector spaces are introduced only very briefly, but it is gratifying to see that Chapter 8 covers not only the Kreĭn-Milman theorem on compact convex subsets of a locally convex vector space, but also Choquet's theorem for such sets at least in the metrizable case. Unfortunately, the relevance of this important result remains a bit unclear, since for examples and applications the reader is referred to the book by R. R. Phelps [*Lectures on Choquet's theorem*, second edition, Lecture Notes in Math., 1757, Springer, Berlin, 2001; MR1835574].

With more than 600 pages, the book under review is about one half longer than each of the two books by Conway and Rudin. Thus, even with well prepared and highly motivated students, it seems almost impossible to cover the entire material of this book in a typical one-year course in functional analysis. The authors are very well aware of this and encourage both the student and the lecturer to be brave enough to jump over topics and pick the material of most interest. A diagram called *Leitfaden* at the end of the preface on the interdependence of the chapters should be helpful to design such an individual course.

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