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BOOK REVIEW

Heights of Polynomials and Entropy in Algebraic Dynamics

By GRAHAM EVEREST and THOMAS WARD
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This monograph explores the connection between Mahler measure, elliptic height, and entropy of certain dynamical systems associated with Noetherian modules and elliptic curves.

The first of these notions, *Mahler measure*, plays an important role in the study of actions of algebraic $\mathbb{Z}^d$-actions, i.e. of $\mathbb{Z}^d$-actions by automorphisms of compact abelian groups: it yields an 'entropy formula' for all such actions which extends the classical entropy formula for toral automorphisms. This formula, established by Lind, Schmidt and one of the authors of this monograph (Ward), is a far-reaching generalization of the classical entropy formula for ergodic toral automorphisms.

Apart from the entropy formula itself, there are many further connections between Mahler measure and algebraic dynamical systems. For example, *Lehmer's conjecture* concerning a possible gap in the range of Mahler measure is equivalent to the question of whether there exist ergodic automorphisms of the infinite-dimensional torus with finite entropy, as was pointed out by Lind. Lawton's result on approximation of higher-dimensional Mahler measure by one-dimensional Mahler measure is equivalent to a statement about approximating entropy of algebraic $\mathbb{Z}^d$-actions by entropies of certain embedded $\mathbb{Z}$-actions.

The first four chapters of the text are devoted to a detailed discussion of Mahler measure, including many examples and historical remarks. Although the material does not go significantly beyond that covered in the corresponding sections of the monograph 'Dynamical systems of algebraic origin' by the reviewer, the account given here is more leisurely and less condensed, and therefore better suited as an introduction to algebraic $\mathbb{Z}^d$-actions.

The final chapters, Chapters 5 and 6, deal with *Elliptical heights* and *Elliptical Mahler measure*, and start with a brief and down-to-earth introduction to elliptic curves. However, the connection between elliptic Mahler measure and dynamical systems remains somewhat tenuous in spite of the authors' efforts, since there appears to be no very satisfactory class of dynamical systems associated with elliptic curves.

In conclusion, the material of the book is very well presented and the historical references, examples and exercises make it a useful text for anybody trying to get into the spirit of algebraic dynamical systems. The connection between elliptic curves and dynamics could be described as food for thought rather than a subject for a monograph, but makes, nevertheless, interesting reading.

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