**Aka, Menny**; **Einsiedler, Manfred**; **Ward, Thomas**

**A journey through the realm of numbers. From quadratic equations to quadratic reciprocity.**

(English) Zbl 07241692

Springer Undergraduate Mathematics Series. Cham: Springer (ISBN 978-3-030-55232-9/pbk; 978-3-030-55233-6/ebook). xix, 344 p. (2020).

This book offers a leisurely path to elementary number theory accessible to bright and motivated high-school students. The first chapter deals with the solution of cubic equations and complex numbers, the second with Cantor's diagonal argument. Chapter 3 introduces the problem of writing integers as sums of squares, and Chapter 4 gives the beautiful windmill proof of Fermat's Two-Squares Theorem (it would have been nice to include the reference to Alexander Spivak's article on *Sums of Squares* (in Russian)) based on Zagier's one-line proof. Chapter 5 is about abstract rings (Euclidean rings, unique factorization); in Chapter 6 a few diophantine equations are solved with the help of some quadratic rings with unique factorization. In Chapter 7 we find a proof that the multiplicative group of the finite field with $p$ elements is cyclic, together with applications to one-way functions in cryptography. The final chapter presents Eisenstein's geometric proof of the quadratic reciprocity law plus a few results on the arithmetic of the ring $\mathbb{Z}[\sqrt{2}]$.

There is also a section that introduces the readers to Sage, and lots of exercises with hints. There are thousands of books out there that popularize mathematics by removing the mathematics from the text; the present book is different: it popularizes number theory and keeps the mathematics in. It is clearly written, suitable for self study, and it deserves a wide readership.

Reviewer: Franz Lemmermeyer (Jagstzell)

**MSC:**

00A09 Popularization of mathematics
11-01 Introductory exposition (textbooks, tutorial papers, etc.) pertaining to number theory
11Dxx Diophantine equations
97Fxx Education of arithmetic and number theory
97Hxx Algebra education

**Keywords:**

number theory; congruences; sums of squares; quadratic reciprocity; diophantine equations

**Software:**

OEIS; PARI/GP; Python; SageMath

**Full Text:** DOI