Zentralblatt MATH

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Ergodic theory. With a view towards number theory.

Graduate Texts in Mathematics 259. London: Springer (ISBN 978-0-85729-020-5/hbk; 978-0-85729-021-2/ebook). xvii, 481 p. EUR 59.95/net; £ 53.99; SFR 86.00 (2011).

The book contains a presentation of the ergodic theory field, focusing mainly on results applicable to number theory.

The recognition of the authors as outstanding researchers in the domain, the current interest of the approached area and the comprehensive bibliography (almost 400 references) are only some immediate indications of the book's value.

The work presents those mechanisms of ergodic theory that connect it to other fields, such as arithmetic, geometry, probabilistic methods etc. and especially number theory, making it also of interest for researchers, specialists, professors and students that work within some other areas than precisely ergodic theory.

Even though it contains 3 appendices on Measure Theory, Functional Analysis and Topological Groups that are supposed to familiarize the reader with the basics notions, definitions and results, it is not an easy book to read without some preliminary knowledge.

Each chapter contains exercises related to the topics discussed in the current section. The book also provides hints for solving some of the proposed exercises.

Getting into more detail, the book is structured into 11 chapters, which are not necessary to be completed in the given order, but according to the reader's interests, as the authors underline from the beginning, in the Leitfaden. The first chapter tries to familiarize us with some particular examples (circle rotation, equidistribution of polynomials, quadratic forms etc.) of the application of ergodic theory with the scope of making the reader aware of the applicability of the theory to some other areas. The second chapter introduces the basic notions and fundamental results of ergodic theory: ergodicity, recurrence, maximal inequality and maximal ergodic theorem that lead to pointwise ergodic theory, strong and weak mixing etc. Chapter 3 treats the decomposition of the real numbers into continued fractions, their basic properties, the relation with an explicit preserving transformation (Gauss map), defines, characterises and exemplifies the badly approximable numbers and the relation between the periodicity of continued fraction expansion and the quadratic irrationals (Lagrange's theorem). Chapter 4 (Invariant Measures for Continuous Maps) continues the introduction to ergodic theory started in Chapter 1 and extends the topological dynamics. It refers to the existence of invariant measures, ergodic decomposition and unique ergodicity. The equidistribution of intervals, generic points and irrational polynomials denote the applicability to number theory. The fifth chapter provides more advanced notions and results from measure theory that are needed in the following chapters: conditional expectations and measures, increasing and decreasing martingales theorems, algebra and maps connections. Chapter 6 (Factors and Joinings) starts a more ambitious approach of the ergodic theory, by using the conditional measures. The ergodic theorem and decomposition are revised according to the notions defined in the previously chapter. Joinings are introduced as a way of determining the similarities of two measure - preserving systems. Chapter 7 (Furstenberg's Proof of Szemeredi's Theorem) provides the latest proof of Szemeredi's theorem and some related results. Chapter 8 presents the ergodic theory for group actions, as a more general approach than the integers or the reals studied in the first chapters. Previous notions and results (measure preserving, ergodic decomposition, ergodic theorem etc.) are extended in order to allow group actions. The next chapter introduces the connection between the Gauss map and hyperbolic surfaces, respectively the geodesic flow. It also provides all the minimal knowledge of Lie theory and differential geometry, necessary to understand the results. Chapter 10 defines the notion of nilrotation as the class of rotations on quotients of nilpotent groups, by studying the rotations of the quotient of the Heisenberg group. The final chapter refers mainly to the ergodicity of the horocycle flow and its properties, in comparison with the geodesic flow discussed in Chapter 9.

As a final opinion, "*Ergodic Theory. With a view toward number theory*" is now an indispensable reference in the domain and offers important instruments of research for other theoretical fields.

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