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Everest, Graham; Ward, Thomas**Heights of polynomials and entropy in algebraic dynamics.** (English)

Universitext. London: Springer. xii, 212 p. DM 99.00; öS 723.00; sFr. 90.00; £35.00; \$ 69.95 (1999). [ISBN 1-85233-125-9/hbk]

This unusual book is based on a course given to postgraduate students at the University of East Anglia. It could serve as a textbook or as interesting supplementary reading for a course in either algebraic dynamics or elliptic curves.

The first four chapters of the book explore the relationship between the classical Mahler measure of a polynomial in one or many variables and certain dynamical systems of algebraic origin. This part of the book could serve as a gentle introduction to the more advanced treatise of *K. Schmidt* [Dynamical systems of algebraic origin, Prog. Math. 128, Birkhäuser, Basel (1995; [Zbl 0833.28001](#))]. For example, in the book under review over 10 pages are devoted to the proof of a special case of Lawton's theorem on the approximation of the Mahler measure of two variable polynomials by the Mahler measure of one variable polynomials, while in Schmidt's book the proof of the complete theorem occupies just over 2 pages. In general, the authors follow the sound pedagogical practice of giving detailed proofs of special cases and providing references to the literature for more general results.

The final two chapters of the text provide a brief introduction to elliptic curves and to the elliptic Mahler measure recently introduced by *G. Everest* and *B. ní Fhlathúin* [Math. Proc. Camb. Philos. Soc. 120, 13-25 (1996; [Zbl 0865.11068](#))]. Here only a brief hint is given of the connection with dynamical systems.

There is an interesting set of exercises in each chapter with an appendix of 23 pages of hints for these exercises. In addition there is a collection of 12 open problems, most of which are quite difficult. For example, problem 3 on page 194 is of the same order of difficulty as proving the existence of infinitely many Mersenne primes. It should be pointed out that problem 1, concerning an elementary proof of Mahler's inequality $M(F') \leq \deg(F)M(F)$ was posed by J. Vaaler in the Problems section of the American Mathematical Monthly and solved by the reviewer using an elementary theorem of Bernstein [Advanced Problem 6613, Am. Math. Mon. 98, 451-452 (1991)]. (Of course students will learn much from Mahler's proof using subharmonic functions which is sketched in Appendix D of the text).

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Keywords : Mahler measure; entropy; algebraic dynamics; elliptic curve; Lehmer's problem; heights of polynomials; dynamical systems; elliptic Mahler measure

Classification :

- *11R09 Polynomials over global fields
- 11-02 Research monographs (number theory)
- 28D05 Measure-preserving transformations
- 11G05 Elliptic curves over global fields

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