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Index of Notation

- π, π_g , representation, 1
- \cdot , group action, 1
- Δ , Laplace operator, 2
- m_G, m , Haar measure on G , 3
- $B(\mathcal{H})$, space of bounded operators on a Hilbert space, 4
- $\mathbb{1}_G$, trivial representation, 4
- λ_g , (left) regular representation, 5
- $g_*\mu$, push-forward of a measure, 6
- supp, support of a function, 7
- ρ_g , (right) regular representation, 9
- Im, image of a map, 12
- $m_G^{(r)}$, right Haar measure on G , 13
- $\pi < \rho$, containment of representations, 15
- \mathcal{H}^∞ , Hilbert space direct sum, 15
- C_n , cyclic group of order n , 17
- C_G , centre of G , 18
- Graph(T), graph of operator, 19
- \mathbb{H} , Hamiltonian quaternions, 21
- D_n , dihedral group, 22
- A_n , alternating group, 24
- $M(G)$, measure algebra of G , 27
- $\pi_*(\nu)$, convolution operator associated to ν , 30
- δ_g , Dirac measure, 31
- $R\int$, Riemann integral, 33
- φ , (often) a matrix coefficient, 34
- $C(G)$, continuous functions on G , 34
- $C_b(G)$, continuous bounded functions on G , 34
- φ_v^π , diagonal (principal) matrix coefficient, 35
- $\varphi_{v,w}^\pi$, (non-diagonal) matrix coefficient, 35
- ϕ , (often) a positive-definite function, 36
- $\mathcal{P}(G)$, positive-definite functions on G , 39
- $\mathcal{P}^1(G)$, normalized positive-definite functions on G , 39
- $\mathcal{P}^{\leq 1}(G)$, convex subset of space of positive-definite functions on G , 47
- SL, special linear group, 54
- \widehat{G} , Pontryagin dual of abelian group G , 59
- \check{f} , Fourier transform of f , 62
- $\sigma(L^1(G))$, space of algebra homomorphisms, 62
- M_g , multiplication representation, 68
- H^\perp , annihilator of subgroup H , 87
- \mathcal{M}^C , space of finite complex-valued measures, 104
- π_{FC} , functional calculus, 108
- μ_{max} , maximal spectral type, 113
- $C(\pi)$, centralizer of representation π , 118
- s_d , standard polynomial, 128
- \wr , wreath product, 142
- $G_{>0}$, connected ‘ $ax+b$ ’ group, 153
- G_\times , full affine group in one dimension, 153
- Ind_H^G , induced representation, 167
- Sol, the solvable group $\mathbb{R} \ltimes \mathbb{R}^2$, 168
- R_ξ , rotation map, 179
- $\pi_{\mu,c}$, unitary representation defined by a cocycle, 180
- \mathcal{B}_X^A , σ -algebra of fixed sets under an action, 174
- \mathcal{P}_π^1 , normalized positive-definite functions associated to a representation, 196
- $\pi \prec \rho$, weak containment of representations, 197
- $CO(\cdot)$, neighbourhood in the compact-open topology, 208
- $\mathcal{FO}(\cdot)$, principal Fell open set, 208
- \otimes , tensor product, 221
- \otimes_{la} , linear algebra tensor product, 221

- G^\sharp , space of conjugacy classes, 250
- SU_2 , special unitary group, 261
- Sym^n , space of homogeneous polynomials of degree n , 268
- Ad, adjoint representation of a Lie group, 296
- $\kappa(\pi)$, decay exponent for a representation, 312
- \mathbb{H} , upper half-plane model, 319
- \mathbb{D} , disk model, 321
- p_π , integrability exponent, 330
- δ_n^- , holomorphic discrete series representation, 335
- $H(\mathbb{D})$, Hardy space, 340
- $\|f\|_{H(D)}$, Hardy norm, 340
- Δ_G , modular character on G , 345
- $\text{Ind}_B^G(\pi_B)$, induced representation, 347
- $\hat{=}$, corresponds to, 360
- $Sp_4(\mathbb{R})$, symplectic group in four dimensions, 360
- \mathcal{R} , set of roots, 363
- $\mathcal{H}(q)$, hyperbola modulo q , 383
- $K_{a,b}^q$, Kloosterman sum, 385
- $X(N)$, congruence quotient, 391
- $SO(2)$, special orthogonal group, 405
- \mathcal{H}^G , subspace of invariant vectors in a unitary representation, 419

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