

“An introduction to number theory” by Everest and Ward
errata file (items marked * are corrected in June 2006 reprint)

Please send comments or corrections to `t.ward@uea.ac.uk`

p. 12, equation (1.4): In fact the proof shows the slightly stronger inequality $\sum_{p \leq N} \frac{1}{p} > \log \log N - 1$.

*p. 19, just after equation (1.12): “We now prove the inequality (1.11) by induction”

p. 20, line 10: We need to have $n \geq 5$ here because a few lines later the inequality $\frac{4}{9}n^2 > 2n$ is used.

p. 20, line 11: The inequality should be $p \leq n$ rather than $p < n$.

p. 21, equation (1.17): The term $\frac{4}{3} \log 2$ should be $\frac{4}{3}n \log 2$.

p. 26, Example 1.14: $23n + 1$ is also prime for $n = 6$, which should therefore be added to the displayed list of numbers.

*p. 48, line -4: Displayed equation should read

$$(-1)(-2) \cdots (-2n)(2n)(2n-1) \cdots 3 \cdot 2 \cdot 1 = (2n!)^2 (-1)^{2n} \equiv -1 \pmod{p}.$$

p. 51, before line -10: The possibility that $w = x = y = z = 0$ must be excluded in this argument. Note that the a and b found in Lemma 2.9 may be chosen with $|a|, |b| < p/2$. Then $0 < a^2 + b^2 + 1 < 2(p/2)^2 + 1 < p^2$, so $0 < m < p$. If w, x, y, z are all zero then

$$m^2 |a^2 + b^2 + c^2 + d^2 = mp,$$

giving a contradiction if $m > 1$.

p. 73, Exercise 3.16: The third Legendre symbol should be $\left(\frac{1003}{113}\right)$ (the point is 111 is not prime).

p. 76, line -12: This should read $1 \leq |e| < 1 + 2\sqrt{d}$.

p. 79, proof of Theorem 3.22: This is garbled and should read as follows.

PROOF. Assume that (α, γ) is a primitive integral solution to Equation (3.18). By Theorem 1.23, there are integers β, δ with

$$\alpha\delta - \beta\gamma = 1.$$

Let

$$\begin{bmatrix} x \\ y \end{bmatrix} = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} \begin{bmatrix} X \\ Y \end{bmatrix}.$$

Notice that $\det \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} = 1$, so this matrix is an invertible transformation on \mathbb{Z}^2 . Express Equation (3.18) in the variables X and Y to obtain

$$\begin{aligned} a(\alpha X + \beta Y)^2 + b(\alpha X + \beta Y)(\gamma X + \delta Y) + c(\gamma X + \delta Y)^2 \\ = nX^2 + (2r + \rho)XY + sY^2 = n, \end{aligned}$$

where $s = a\beta^2 + b\beta\delta + c\delta^2$ and

$$2r + \rho = 2a\alpha\beta + 2c\gamma\delta + b(\alpha\delta + \beta\gamma).$$

Notice that s is an integer and $\alpha\delta + \beta\gamma = 1 + 2\beta\gamma$ is odd, so $b(\alpha\delta + \beta\gamma) - \rho$ is even, and

$$r = a\alpha\beta + c\gamma\delta + \frac{1}{2}(b(\alpha\delta + \beta\gamma) - \rho)$$

is an integer also. The equation

$$nX^2 + (2r + \rho)XY + sY^2 = n$$

has the solution $X = 1, Y = 0$ corresponding to

$$\begin{bmatrix} \alpha \\ \gamma \end{bmatrix} = \begin{bmatrix} \alpha & \beta \\ \gamma & \delta \end{bmatrix} \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

The discriminant is unchanged, so

$$(2r + \rho)^2 - 4sn = \Delta,$$

so

$$r^2 + \rho r - \left(\frac{\Delta - \rho}{4}\right) = sn,$$

showing that r is an integer solution of the congruence (3.19).

The end of the proof should have the displayed equation:

$$nx^2 + (2r + \rho)xy + sy^2 = n.$$

p. 87, line 11,12: This should read $\alpha = c + g\delta$ and $\beta = h$.

p. 96, Exercise 5.2: $(1, 0)$ is not on the curve – this should be $(0, 0)$.

p. 101, line 12: This should read “Let T^2 denote the denominator of v ;

p. 115, line 5: “By the strong form of Siegel’s Theorem (Theorem 7.13)...”

p. 135, Lemma 7.4: The definition of morphism needs to say there is no common zero over the algebraic closure of the rationals, and this condition then needs to be checked where Lemma 7.4 is used later [thanks to Morten Hein Tiljeset for pointing this out]

p. 171, Example 8.19: Summations should start at $n = 0$ throughout.

p. 178, Exercise 8.23: The equation in (c) should read

$$B(s) = 1 + \frac{1}{2^s} - 2 \cdot \frac{1}{3^s} + \frac{1}{4^s} + \frac{1}{5^s} - 2 \cdot \frac{1}{6^s} + \cdots .$$

p. 191, line -9: range of integration should be from y to $y + 1$.

p. 192, equation (9.8): This should be

$$-c_k + c_{-k} = 2i \int_0^1 g(x) \sin(2\pi kx) dx \rightarrow 0 \text{ as } k \rightarrow \infty.$$

Equation (9.9): $\sin(2K + 1)x$ should be $\sin((2K + 1)\pi x)$.

p. 193, line -10: The right-hand side should be just $G_N(x)$.

p. 195, line -4: This should read “uniformly and absolutely”.

p. 204, Exercise 9.11: The reference should be to Exercise 8.24 not equation (8.24).

*p.206, line 6: “The disproof of Mertens’ conjecture”

p.209, equation (10.6): The right-hand side should be $\sum_{n=0}^{\infty} (-t^2)^n$.

p. 221, line -3: Lower limit of summation should be $\nu = 0$.

p. 249, line -1: A factor of $C_1(k)$ should appear at the end of the expression.