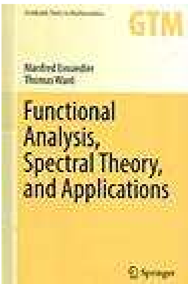
MATHEMATICAL ASSOCIATION OF AMERICA

Functional Analysis, Spectral Theory, and Applications



Manfred Einsiedler and Thomas Ward

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[Reviewed by Mark Hunacek, on 01/19/2018]

This is an attractive new textbook in functional analysis, aimed at, I would say, second-year (or well-prepared first-year) graduate students. A solid undergraduate-level background in linear algebra and real analysis is an essential prerequisite; some familiarity with complex analysis would also be helpful. Measure theory is also used (e.g., the L^p spaces are introduced early and used throughout), but the basics of this topic are summarized in an appendix; another appendix also reviews those aspects of topology that are used in the text. As is often the case with appendices like these, however, they serve best as a review of material already studied, rather than as a place to learn the material for the first time.

The authors begin their Preface by asking what the rationale could be for “yet another book on functional analysis” and then give what I think is an unduly modest answer: “Little indeed can justify this beyond our own enjoyment of the beauty and power of the topics introduced here.”

I must respectfully disagree. Two obvious additional justifications for this book are (a) the selection and organization of the topics, and (b) the expository skill of the authors.

With regard to (a): there is a lot of material covered here, certainly enough (in fact, more than enough) to fill out a full year’s course in functional analysis. The material covered includes, of course, all the familiar topics that one would expect to see in any introductory course of this nature: Banach and Hilbert spaces are introduced early (chapters 2 and 3) proofs and applications of the Uniform Boundedness and Open Mapping theorems immediately follow (chapter 4); dual spaces and the Hahn-Banach theorem come a few chapters later (chapter 7), but can be covered right after the second and third chapters. But there is also much more. In addition to these core topics, Banach algebras are covered (chapter 11) and applied to the spectral theorem and functional calculus (chapter 12); there are also chapters on the spectral theory of unbounded self-adjoint operators (chapter 13), and locally convex vector spaces and convexity (chapter 8). This latter chapter includes a brief look at distributions and a statement and proof of the Krein-Millman theorem.

Moreover, in keeping with the title, the book devotes more attention than is customary to applications, both to analysis and other areas of mathematics. Some of these applications are definitely not standard for books at this level. There are discussions of applications of functional analysis, for example, to partial differential equations (e.g., chapter 5 discusses applications of Sobolev spaces to boundary value problems, and chapter

6 discusses eigenfunctions of the Laplace operator), unitary group representations (chapter 3), and even the prime number theorem, proved (following Terry Tao) in chapter 14 using Banach algebras and Fourier analysis. In addition, chapter 10 discusses amenable groups and provides applications to graph theory.

The authors have done a good job of organizing the material so as to achieve considerable flexibility in the use of the text. There is a useful dependency chart ("Leitfaden") that illustrates a number of different ways to select a path through the text for either a one-semester or one-year course. In particular, the instructor can easily decide to either emphasize or downplay the applications, as desired.

As for point (b) above, the authors write well and take pains to motivate discussions whenever possible. The first chapter of the book, for example, is *entirely* motivational; it introduces a number of applications that functional analysis relates to, and also gives a one-page overview of what spectral theory is all about, tying it in to finite-dimensional linear algebra as motivation. The authors state that the "reader may, and the lecturer should, skip this chapter or return to it later, as convenient."

The authors' concern with student understanding extends to their writing style, which is very clear and generally accompanied by a good supply of examples. There are also exercises interspersed throughout the text, hints or solutions to about half of which can be found in the back of the book. A good bibliography ends the text.

There are also a number of footnotes scattered throughout the text. In an unusual feature, there are two kinds of them. Numbered footnotes refer the reader to a section titled "Notes" at the end of the book, and footnotes marked with a dagger symbol appear at the bottom of the page.

To summarize and conclude: the large amount of material covered in this book, including some fairly unusual topics, as well its overall readability, makes it useful as a reference as well as a potential graduate textbook. If you like functional analysis, teach it, or use it in your work, this book certainly merits a careful look.

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