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★**Recurrence sequences.**

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Recurrence sequences appear almost everywhere in mathematics and computer science. Their study is, as well, plainly of intrinsic interest and has been a central part of number theory for many years. Surprisingly enough, there was no book in the literature entirely devoted to recurrence sequences: only some surveys, or chapters of books, dealt with the subject. With the book under review, the authors fill this gap in a remarkable way. Its content can be summarized as follows.

In Chapters 1 to 8, general results concerning linear recurrence sequences are presented. The topics include various estimates for the number of solutions of equations, inequalities and congruences involving linear recurrence sequences. Also, there are estimates for exponential sums involving linear recurrence sequences as well as results on the behaviour of arithmetic functions on values of linear recurrence sequences. Apart from basic results from the theory of finite fields and from algebraic number theory, there are three important tools.

A first one is p-adic analysis, which is used in the proof of the Skolem–Mahler–Lech theorem. This theorem asserts that the set of zeros of a linear recurrence sequence $(u_n)_n$ (that is, the set of indices n for which $u_n = 0$) over a field of characteristic zero comprises a finite set together with a finite number of arithmetic progressions. Also, p-adic analysis is at the heart of the proof of the Hadamard quotient problem, and may in some cases be applied to get very good estimates for the number of solutions of equations.

A second tool is Baker's theory of linear forms in the logarithms of algebraic numbers. It yields effective growth rate estimates (under some restrictions) and many arithmetical results: lower bounds for the greatest prime factor of the n-term of a linear recurrence sequence $(u_n)_n$, finiteness of the number of perfect powers in $(u_n)_n$ (under some restrictions), etc.

A third tool is the Schmidt subspace theorem, and in particular its applications to sums of S-units. Specifically, linear recurrence sequences provide a special case of S-unit sums. As an example of a striking result obtained thanks to this theory, it has been proved by H. P. Schlickewei and W. M. Schmidt [Compositio Math. **120** (2000), no. 2, 193–225; MR 2001b:11022] that the number of zeros in any nondegenerate linear recurrence sequence of order n over a number field of degree d is at most equal to $(2n)^{35n^3} d^{6n^2}$.

In Chapters 9 to 14, a selection of applications are given, together with a study of some special sequences. Chapter 10 deals with elliptic divisibility sequences, an area with geometric and Diophantine methods coming to the fore. Other applications include graph theory, dynamical systems, pseudo-random number generators, computer science, and coding theory.

Rather than giving complete proofs, the authors often prefer to point out the main arguments of the proofs, or to sketch them. But for every result not proved in their book, they give either a direct reference or a pointer to an easily available survey in which a proof can be found.

With its 1382 bibliographical references, this well-written book will be extremely useful for anyone interested in any of the many aspects of linear recurrence sequences. *Yann Bugeaud* (F-STRAS)